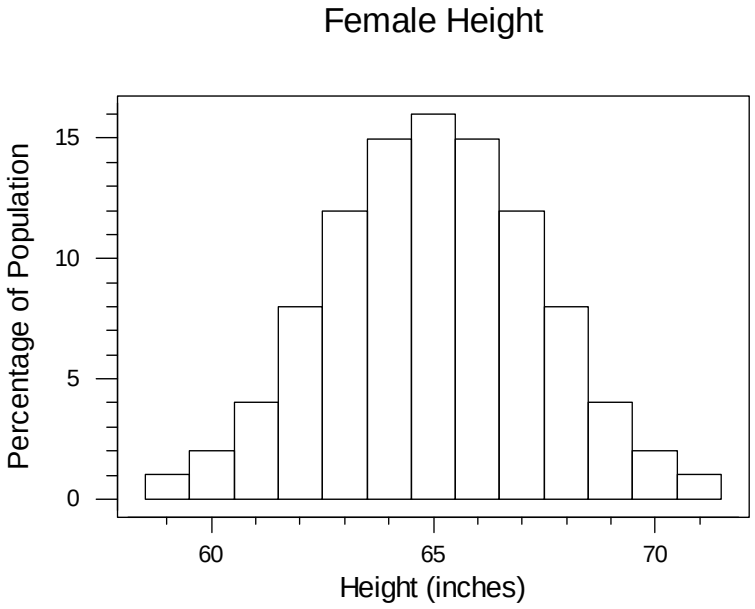
Normal distribution NAME:

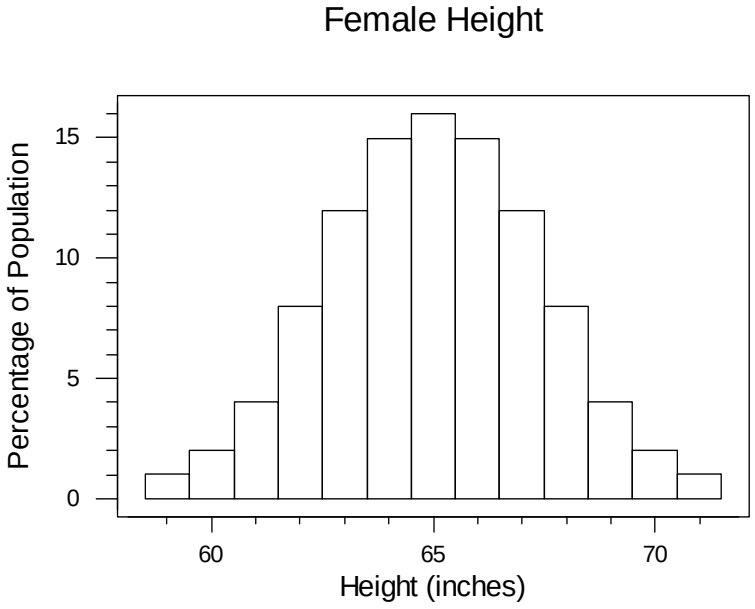
Consider the histogram below detailing female height. The mean is 65 inches and the standard deviation is 2.5 inches. We will use it to introduce and practice the ideas of normal distributions.



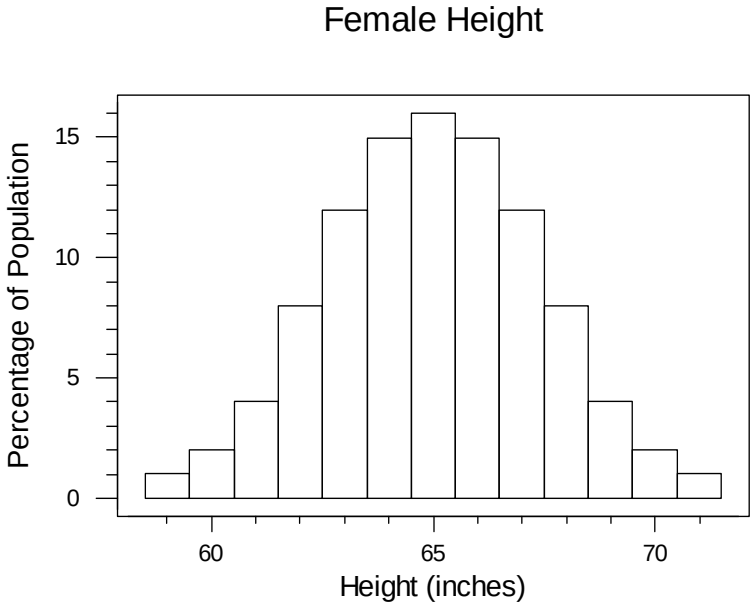
1. Notice the bars are centered on the whole numbers 59, 60, 61, 62, and so on. So the height of the bar above the 63 tells us the percent of all females who are between 62.5 and 63.5 inches tall. You can round all percentages to the nearest whole numbers.

What percent of females are between 62.5 and 63.5 inches tall? Shade this bar. Since the bar has a width of 1, the area of this bar represents this percentage. We will use this notion throughout our study of normal distributions.

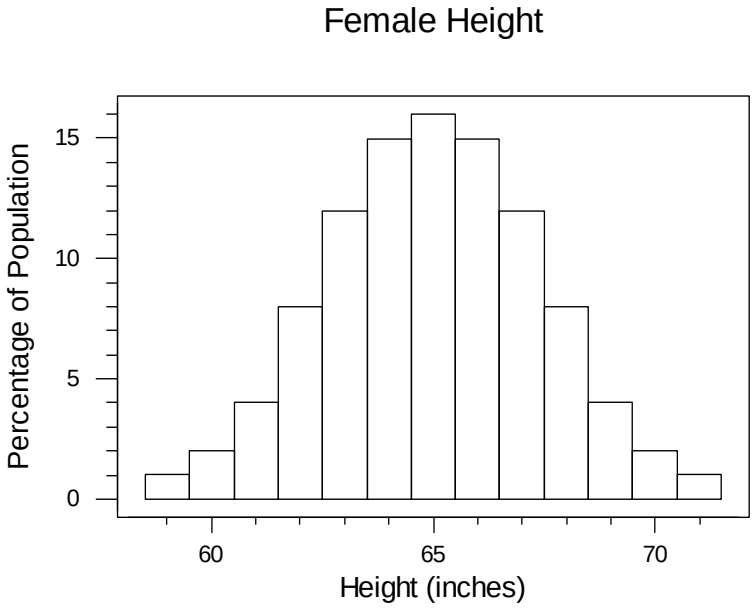
2. What percent of females are between 61.5 and 65.5 inches tall? How do you find this? Shade the corresponding bars on the histogram below. Notice the desired percentage is also the total area of the corresponding bars.



3. What percent of females have a height less than 67.5 inches? How do you find this? Shade the corresponding bars on the histogram below. Notice the desired percentage is also the total area of the corresponding bars.



4. If we draw a point at the center of the top of every bar and connect them in a smooth curve, we get a graph called the **density curve of the normal distribution**. Use the copy of the histogram below to draw in the density curve. Let the left and right tails trail off toward the *x*-axis but *not* touch the *x*-axis. Notice how the density curve shows the distribution of female height just like the histogram does.



5. Let's return to the problem in question 3 that asks to find the percent of females who are less than 67.5 inches tall. Recall you shaded the corresponding bars in the histogram for this question. Notice this area is approximated by the area under the normal curve to the left of 67.5. On the graph above, draw a vertical line at 67.5 and then shade the area *under the curve* to the left of 67.5.

Notice this area only **approximates the area** of the bars in the histogram. We now want to figure out the area under the curve to the left of 67.5 (pretending that we did *not* have the histogram). This is the idea of finding percentages (or probabilities) for normal distributions. For the rest of this worksheet and chapter, you will find percentages (areas) this way. Go on to step 6.

6. We would like to use a table to look up the area to the left of 67.5. But we do *not* have such a table. The specific shape of the curve is determined by the mean and standard deviation (in our case, mean 65 and standard deviation 2.5). The curve, and hence the area under the curve, will be different for different distributions; we *cannot* have a table for *every possible* normal distribution.

So we have to change our number, 67.5 (we'll call it an **observation**), into a **standard score**. We then look this standard score up on a table to find the percentage of standard scores that are less than our standard score. That will be the same as the percentage we are after, the percentage of females who are less than 67.5 inches tall.

Basically what this does is reduce our normal distribution with a mean of 65 and a standard deviation of 2.5 into a normal distribution with a mean of 0 and a standard deviation of 1. This special distribution (mean 0, standard deviation 1) is called the **standard normal distribution**. Go to step 7.

7. The formula is $z = \frac{x - \mu}{\sigma}$ where z is the standard score, x is the observation (our value of 67.5 inches, in this case), μ is the mean of the original distribution, and σ is the standard deviation of the original distribution. Find the standard score for our observation of 67.5. (Recall the mean is 65 and the standard deviation is 2.5.) Show work.

Notice this essentially tells you how many standard deviations above or below the mean that the observation is. (Did you get z = 1.00? That means 67.5 is 1 standard deviation above the mean of 65. Does that sound right?)

8. Now we'll look this standard score up on the table in the back of the book, which is why I rounded it to two decimal places. The table tells you the **percentile** that corresponds to the standard score; that is, it tells you the percent of the population that has a standard score below the observation. In terms of the situation, it tells you the percent of females who have a height less than 67.5 inches.

Find 1.0 in the leftmost column (labeled z) of the table and then find the column labeled .00. The table entry for the row 1.0 and the column .00 tells you the percentage of females whose heights are less than 67.5 inches. What is that percentage? Does that match what you found in question 3, when you added up the bars of the histogram? Why do you think there is a slight difference?

Let's try a few more. Round your standard scores to two decimal places. Do *not* round the final percentages.

9. Find the percentage of females whose heights are less than 65.5 inches. (Find the standard score and look it up in the table.) Show work. Include a quick normal curve (without underlying histogram) with the appropriate area shaded.

10. Find the percentage of females whose heights are less than 61.5 inches? (Mind the sign of the standard score! Notice 61.5 is *below* the mean of 65, so the standard score should be negative.) Show work. Include a quick normal curve (without underlying histogram) with the appropriate area shaded.

11. Find the percentage of females whose heights are *between* 61.5 and 65.5 inches. You can use the answers for #9 and #10 to find this. Does this approximate your answer for question 2? Include a quick normal curve (without underlying histogram) with the appropriate area shaded.