Page 305 Example 5 without t substitution

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-3x}\right) = \infty - \infty$$ (verify limit $=\infty-\infty$)

$$\lim_{x\to\infty}\left(x-\sqrt{x^2}\left(\sqrt{1-3/x}\right)\right)$$ (preparing to factor out x from both terms)

$$=\lim_{x\to\infty}\left(x-x\left(\sqrt{1-3/x}\right)\right)$$ ($\sqrt{x^2}=x$)

$$=\lim_{x\to\infty} x\left(1-\sqrt{1-3/x}\right)$$ (factored x)

Nutty $x=\frac{1}{1/x}$

$$=\lim_{x\to\infty}\frac{1-\sqrt{1-3/x}}{1/x} = \frac{0}{0}$$ (evaluated limit and see it's indeter. 0/0)

$$=\lim_{x\to\infty}\frac{1-(1-3x^{-1})^{1/2}}{x^{-1}}$$ (prepare to take derivative

$$=\lim_{x\to\infty}\frac{-\frac{1}{2}(1-3x^{-1})^{-1/2}(3\cancel{x^{-2}})}{-\cancel{x^{-2}}}$$

L'Hopitals Rule (took derivatives of top and bottom)

$$=\lim_{x\to\infty}\frac{3}{2\left(1-3/x\right)^{1/2}} = \frac{3}{2}$$ (direct sub. to find limit. and dance, dance, dance)

$3/x \to 0$

(52) $$\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{(x-1)^{1/2}}\right) = \infty - \infty$$

$$= \lim_{x\to 1^+}\left(\frac{(x-1)^{1/2}-(x-1)}{(x-1)^{3/2}}\right)$$ ✓ (got LCD and combined fractions)

L'Hop

$$= \lim_{x\to 1^+}\left(\frac{\frac{1}{2}(x-1)^{-1/2}-1}{\frac{3}{2}(x-1)^{1/2}}\right)$$ ✓ (took derivatives of top & bottom) L'Hop

$$= \lim_{x\to 1^+}\frac{\frac{1}{2(x-1)^{1/2}}-1}{\frac{3}{2}(x-1)^{1/2}}$$ ✓ (rewrote top)

(work shown below to simplify top)

$$= \lim_{x\to 1^+}\frac{1-2(x-1)^{1/2}}{\cancel{2}(x-1)^{1/2}\,\frac{3}{\cancel{2}}(x-1)^{1/2}}$$

$$= \lim_{x\to 1^+}\frac{1-2(x-1)^{1/2}}{3(x-1)} = \frac{1}{0} = \infty$$ (cancelled) (direct sub for limit) (celebrate)

Scratch work

$$\frac{1}{2(x-1)^{1/2}} - \frac{2(x-1)^{1/2}}{2(x-1)^{1/2}}$$

$$= \frac{1-2(x-1)^{1/2}}{2(x-1)^{1/2}}$$

(#52 work) for steps 3 to 4

$\log_e 1 = 0$
$e^0 = 1$

$\frac{1}{x}$

(59) $\lim_{x \to 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right) = \infty - \infty$ (Notice it's of the $\infty - \infty$ form)

(Got LCD and combined into single fraction)

$= \lim_{x \to 1^+} \frac{\ln x - (x-1)}{(x-1)\ln x} = \frac{0}{0}$ $\qquad \frac{0-0}{0 \cdot 0}$

Now, it's in the indeterminate form 0/0.

So, do derivatives on top and bottom

L'Hop

x^{-1}

$= \lim_{x \to 1^+} \frac{\frac{1}{x} - 1}{1 \ln x + \underbrace{(x-1)\frac{1}{x}}_{\text{simplify this}}} = \lim_{x \to 1^+} \frac{\frac{1}{x} - 1}{\ln x + \underline{1 - \frac{1}{x}}}$

$= \frac{0}{0}$ (still $\frac{0}{0}$!)

L'Hop L'Hop Again

$= \lim_{x \to 1^+} \frac{-x^{-2}}{\frac{1}{x} + x^{-2}}$ find derivatives $= \lim_{x \to 1^+} \frac{-1}{x^2\left(\frac{1}{x} + \frac{1}{x^2}\right)}$ (Change to positive exponents to see better)

$= \lim_{x \to 1^+} \frac{-1}{(x+1)}$ simplify $= -\frac{1}{2}$

reevaluate limit by direct substitution (Yahoo!)