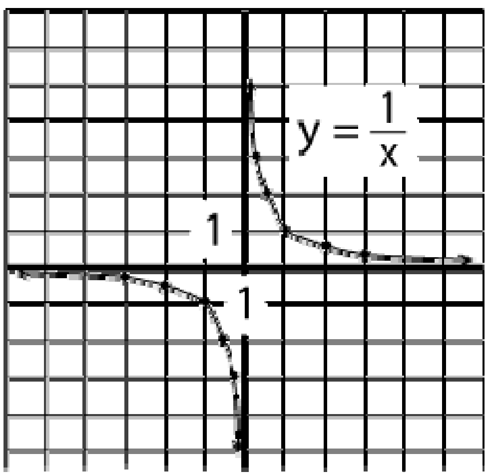
Limits as x approaches infinity 2 NAME:

This worksheet will work on the notion of the limit of a function as x approaches negative or positive infinity. You can think of this limit as the one value the y values approach as we get further and further toward the left or right end of the graph. Remembering horizontal asymptotes of rational functions may be helpful; they will be referred to later.

1. Consider the graph of $y = \frac{1}{x}$ below. Pay particular attention to the far left and far right ends of the graph. What number are the y values getting closer and closer to? The answer is the same on both ends of the graph.



As you go further and further toward the left end of the graph, we say x approaches negative infinity. The number that the y values are approaching at this end of the graph will be called $\lim_{x \to -\infty} \frac{1}{x}$. What is that y value?

Likewise, as you go further and further toward the right end of the graph, we say x approaches positive infinity. The number that the y values are approaching at this end of the graph will be called $\lim_{x \to \infty} \frac{1}{x}$. What is that y value?

Horizontal asymptotes of rational functions essentially use this same idea. In the case of rational functions $f(x)$, the $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$ will be the same number (if it is, indeed, a number). To figure horizontal asymptotes of rational functions, and therefore these limits, recall the following.

A rational function is a function that can be written as a fraction where the top and bottom are both polynomial functions. (Recall that the degree of a polynomial function is the highest exponent of the x's. The leading term of a polynomial is the term that contains this highest exponent.)

There are three cases:

1. degree on top is **equal to** degree on bottom,
2. degree on top is **less than** degree on bottom, and
3. degree on top is **greater than** degree on bottom.

For case 1, the horizontal asymptote is gotten by dividing the leading terms on top and bottom. For case 2, the horizontal asymptote is always $y = 0$. The function $y = \frac{1}{x}$ is an example of this. For case 3, the (oblique) asymptote is the quotient gotten by dividing the bottom into the top. (Technically, this is only an oblique asymptote if the degree on top is one more than the degree on bottom. Otherwise, it is a backbone.) In this last case, we would say the limit does *not* exist, since the y values will *not* approach a single value. However, we will write the limit as positive or negative infinity.

When we are finding limits, we will run into rational functions and have to determine which of these three cases apply.

Find the limits listed below. Try to think analytically but you could also graph the functions and see what value the y values approach as x approaches negative or positive infinity, whichever is indicated by the limit.

2. $\lim_{a\to-\infty} \frac{1}{2(a-2)^2}$

3. $\lim_{x\to\infty} \frac{x^2}{3x^2}$

These next limits involve transcendental functions. It helps to have pictures of the basic function $y = e^x$ and its inverse $y = \ln x$ in your head.

4. $\lim_{x \to -\infty} e^{2x}$

5. $\lim_{x \to \infty} \left(\ln(x+2) + 3 \right)$

6. $\lim_{x \to \infty} \left(\frac{3}{e^x} \right)$

7. $\lim_{x \to -\infty} \left(\frac{3}{e^x} \right)$

8. $\lim_{x \to -\infty} \left(\frac{e^x}{3x+4} \right)$